

CERN-TH/98-382
CPH-S691.1198
LPHE-ORSAY 98/70
ROM2F-98/41
hep-th/9812118

Partial breaking of supersymmetry, open strings and M-theory

I. Antoniadis^a, G. D'Appollonio^b, E. Dudas^{c,d} and A. Sagnotti^e

^a *Centre de Physique Théorique (CNRS UMR 7644), Ecole Polytechnique, F-91128 Palaiseau*

^b *Dipartimento di Fisica, Univ. dell'Aquila, INFN-LNGS, 67010 Coppito, Italy*

^c *TH-Division, CERN, CH-1211 Geneva 23, Switzerland*

^d *LPHE[†], Bât. 211, Univ. Paris-Sud, F-91405 Orsay*

^e *Dipartimento di Fisica, Università di Roma "Tor Vergata"*

INFN, Sezione di Roma "Tor Vergata"

Via della Ricerca Scientifica 1, 00133 Roma, Italy

Abstract

We study total and partial supersymmetry breaking by freely acting orbifolds, or equivalently by Scherk-Schwarz compactifications, in type I string theory. In particular, we describe a four-dimensional chiral compactification with spontaneously broken $N = 1$ supersymmetry, some models with partial $N = 4 \rightarrow N = 2$ and $N = 4 \rightarrow N = 1$ supersymmetry breaking and their heterotic and M-theory duals. A generic feature of these models is that in the gravitational sector and in the spectrum of D-branes parallel to the breaking coordinate, all mass splittings are proportional to the compactification scale, while global (extended) supersymmetry remains unbroken at tree level for the massless excitations of D-branes transverse to the breaking direction.

*Research supported in part by the EEC under TMR contract ERBFMRX-CT96-0090.

[†]Laboratoire associé au CNRS-URA-D0063.

1. Introduction

In a recent paper [1] we started a systematic investigation of type I string vacua where supersymmetry is spontaneously broken by the compactification¹. Aside from their clear potential interest for phenomenology, the resulting models are additional examples of the link between closed and open models [3]-[6]. They may be constructed in a systematic fashion associating suitable D-brane (open string) sectors to projected bulk (closed string) models where supersymmetry breaking is induced by a Scherk-Schwarz (SS) deformation [7]-[11]. In Field Theory, the SS mechanism is realized if all fields are periodic along a compactified direction only up to discrete R-symmetry transformations. In String Theory, the resulting constructions are equivalent to the freely acting orbifolds discussed in [12], where the discrete symmetry is combined with a shift δ along the compact direction.

In the simplest case of toroidal compactification, the shift δ is a fraction of a lattice vector determined by the order of the discrete symmetry (*e.g.* $N\delta = 1$ for Z_N). For closed strings, there are two distinct choices for the shift, depending on whether it acts identically or oppositely on left and right movers, that are related by T-duality transformations ($R \rightarrow 1/R$ for a circle of radius R)². The SS deformation is then recovered multiplying (or dividing) the radius R by N , and amounts to shifting the momenta (or the windings) of all states according to their R-charges. In the following we will refer to these two cases as momentum and winding shifts. The two choices lead to very different open string spectra [1]. Consider, for instance, a D-brane parallel to the direction used for supersymmetry breaking. With momentum shifts, all mass splittings are proportional to $1/R$, with R the compactification radius, and to the discrete charges, as in the closed string sector. Supersymmetry is then restored only in the large radius limit. On the other hand, with

¹Previous work on supersymmetry breaking in open strings is described in [2].

²We consider only the case of light-like Narain vectors, that lead to symmetry restoration in the large (or small) radius limit.

winding shifts all mass splittings vanish for the massless excitations of the D-brane, while supersymmetry is restored in the small radius limit. The physical reason for this result can be better appreciated in the T-dual (type I') picture. In this case, the D-brane is orthogonal to the direction of supersymmetry breaking, that again disappears in the decompactification limit. Therefore, it is naturally insensitive to the breaking of supersymmetry, as long as one does not consider the modes, much heavier than the string scale, that wrap around the compact space. On the other hand, when these modes are excited, the D-brane does feel the breaking, since it effectively extends into the compact space.

This phenomenon is apparently a generic feature of type I compactifications. Therefore, it is instructive to relate these models to heterotic compactifications using string dualities [13, 14]. This is possible since adiabatic arguments [12] justify the use of duality transformations for freely acting orbifolds. Momentum shifts in D9 branes may then be related to ordinary SS compactifications of the weakly coupled $SO(32)$ heterotic string. On the other hand, winding shifts may be related to SS deformations involving the eleventh dimension of M-theory, a non perturbative phenomenon in the $E_8 \times E_8$ heterotic string [15, 16].

In this work we explore further this line of investigation. We thus present a first instance of a four dimensional (4d) $N = 1$ chiral model with D9 and D5 branes, where supersymmetry breaking is induced by a SS deformation based on a Z_2 symmetry, the spacetime fermion parity $(-1)^F$. Our starting point is the six-dimensional supersymmetric model of [5, 17], deformed as in [1], but here we couple the spacetime fermion parity to the winding modes, in order to make the connection with M-theory more transparent. With this choice, the massless states related to the D9 brane are not affected at tree level.

In a T-dual type-I' picture, where the D9 and D5 branes have turned into D8 and D6 branes, supersymmetry is restored in the large radius limit. Actually, this mechanism can be realized only if the 8-branes are equally distributed among the two fixed points of the corresponding orientifold, so that the D8 gauge group becomes a direct product of

two identical (chiral) factors with no direct communication at the massless level. This is similar to the type I' situation in 9 dimensions, where one can take the large radius limit at weak string coupling only if the branes are evenly distributed, so that the gauge group is $SO(16) \times SO(16)$ [13]. This limit then corresponds to the decompactification of the 11th dimension of M-theory, once the two sets of D8 branes are associated with the two 10d walls containing the two E_8 factors [14]. In the type I string, the problem manifests itself as a tadpole generated by the collapse of superheavy winding states that propagate in the transverse channel and become massless in the small radius limit. It is remarkable that these tadpole cancellations single out uniquely the M-theory setup [1].

A peculiarity of our chiral models with supersymmetry breaking induced by $(-1)^F$ is a relative chirality flip, after the SS deformation, between pairs of gauge group factors for branes perpendicular to the breaking direction and located at different fixed points of the orientifold. This phenomenon was already observed in 6 dimensions [1], and is clearly induced by the modified projection of the closed sector. However, we do not have a simple geometrical picture for it, and we are not sure whether it occurs in all similar constructions.

As a second step, we study the effects of SS deformations induced by ordinary R-symmetries, rather than by the fermion parity $(-1)^F$. These R-symmetries are discrete remnants of internal rotations in the 6d compact space and, as explained above, the resulting deformations are equivalent to freely acting orbifolds where internal rotations are combined with shifts [11]. Unlike the case of $(-1)^F$, R-symmetries can break only part of the supersymmetries, typically one half of them, thus yielding examples with partial supersymmetry breaking. In this paper we work out some explicit examples of $N = 4$ type I string compactifications with partial supersymmetry breaking to $N = 2$ or $N = 1$, using Z_2 shifts. Again, we find two distinct options, corresponding to momentum or winding shifts, with D-branes parallel or orthogonal to the breaking direction, and extended supersymmetry partly broken or unbroken for the massless excitations.

This paper is organized as follows. In Section 2 we review briefly the 6d chiral model of ref. [1] and present an $N = 1$ reduction to 4 dimensions, obtained by a Z_6 orbifold compactification with SS supersymmetry breaking induced by $(-1)^F$. In Section 3 we present the simplest example of partial supersymmetry breaking in 5 dimensions, using a Z_2 freely acting orbifold that twists four internal coordinates and shifts the fifth one. We first review the parent type II string model, that breaks partially $N = 8$ to $N = 4$, and then derive its open descendants, where the partial breaking is from $N = 4$ to $N = 2$, in the two cases of momentum (Section 4) and winding (Section 5) shifts.³ In Section 6 we describe a 4d type I model where $N = 4$ supersymmetry is partially broken to $N = 1$ using a $Z_2 \times Z_2$ freely acting orbifold with three different momentum shifts. Finally, Section 7 contains our conclusions. Throughout the paper we use the same conventions as in [1]. Thus, $\alpha' = 2$, and for later use we also define (with η the Dedekind function)

$$Z_{m+a}(\tau) = \frac{q^{\frac{1}{2}(\frac{m+a}{R})^2}}{\eta(\tau)} , \quad \tilde{Z}_{n+b}(\tau) = \frac{q^{\frac{1}{2}((n+b)\frac{R}{2})^2}}{\eta(\tau)} \quad (1.1)$$

while, in relating the direct and transverse channels, we shall use repeatedly the Poisson transformation

$$\sum_m e^{2i\pi mb} Z_{m+a}(-\frac{1}{\tau}) = R e^{-2i\pi ab} \sum_n e^{-2i\pi na} \tilde{Z}_{2n+2b}(\tau) . \quad (1.2)$$

2. A chiral four-dimensional model with $N = 1 \rightarrow N = 0$ breaking

In this Section, using the results of [1], we construct a chiral 4d model with spontaneously broken $N = 1$ supersymmetry, where a portion of the massless spectrum, that after a suitable T-duality may be associated to D9-branes, is still (globally) supersymmetric. The starting point is the 6d $N = 1$ T^4/Z_2 type I orbifold of [5, 17] with all five-branes at the same fixed point, a single 6d tensor multiplet, and a gauge group $U(16)_9 \times U(16)_5$. The

³Here we are counting the number of supersymmetries in terms of 4d Weyl spinors.

open string sector then contains vector multiplets and hypermultiplets in the representations $(\mathbf{120} + \overline{\mathbf{120}}, \mathbf{1})$, $(\mathbf{1}, \mathbf{120} + \overline{\mathbf{120}})$ of the gauge group from the 99 and 55 sectors, and one hypermultiplet in the representation $(\mathbf{16}, \mathbf{16})$ from the 59 sector. Turning on a Wilson line, one can break the D9 gauge group $U(16)_9$ to $U(8)_9 \times U(8)_{9'}$. Then, after a T-duality turning D9 branes into D8 branes and D5 branes into D6 branes, one can interpret the resulting model as M-theory compactified on $T^4 \times S^1/(Z_2 \times Z_2)$. In addition to the $N = 2$ vector multiplets, corresponding to the representations $(\mathbf{64}, \mathbf{1}, \mathbf{1})_R + (\mathbf{1}, \mathbf{64}, \mathbf{1})_R + (\mathbf{1}, \mathbf{1}, \mathbf{256})_R$ of the gauge group $U(8)_8 \times U(8)_{8'} \times U(16)_6$, the resulting massless spectrum contains the hypermultiplets

$$\begin{aligned}
66 & : (\mathbf{1}, \mathbf{1}, \mathbf{120})_L + (\mathbf{1}, \mathbf{1}, \overline{\mathbf{120}})_L , \\
88, 8'8' & : (\mathbf{28}, \mathbf{1}, \mathbf{1})_L + (\overline{\mathbf{28}}, \mathbf{1}, \mathbf{1})_L + (\mathbf{1}, \mathbf{28}, \mathbf{1})_L + (\mathbf{1}, \overline{\mathbf{28}}, \mathbf{1})_L , \\
86, 8'6 & : (\mathbf{8}, \mathbf{1}, \mathbf{16})_L + (\mathbf{1}, \mathbf{8}, \mathbf{16})_L .
\end{aligned} \tag{2.1}$$

Actually, all ND (Neumann-Dirichlet) representations occur as half-multiplets with pairs of conjugate assignments, equivalent after charge conjugation to conventional hypermultiplets. Thus, for instance, the hypermultiplet assigned to the representation $(\mathbf{8}, \mathbf{1}, \mathbf{16})_L$ actually appears in the open spectrum as a pair of half-hypermultiplets associated to the conjugate pair $(\mathbf{8}, \mathbf{1}, \mathbf{16})_L + (\overline{\mathbf{8}}, \mathbf{1}, \overline{\mathbf{16}})_L$.

As shown in [1], starting from the supersymmetric $U(16)_9 \times U(16)_5$ model, one can break 6d supersymmetry completely in a soft fashion by a Scherk-Schwarz deformation along the direction that was T-dualized above (parallel to the D6 branes and transverse to the D8 branes). This is induced by the operator $(-1)^F$ acting on the lattice states, or equivalently by a 2π -rotation in a plane defined by one compact and one non-compact coordinate. Before T-duality, the winding modes are shifted by this operation, while the momentum modes are unaffected. The corresponding world-sheet current anticommutes with the orbifold projection, as required for the consistency of the construction [9, 10]. As in the supersymmetric case, the factorization properties of the amplitudes and the tadpole

conditions uniquely determine the gauge group $U(8)_8 \times U(8)_{8'} \times U(16)_6$, identical to that of the supersymmetric model. The two D8-brane factors live at two different fixed points of the orientifold, and their world-volumes are orthogonal to the compact coordinate responsible for the breaking. The corresponding massless matter content is

$$\begin{aligned}
66 & : \left[(\mathbf{1}, \mathbf{1}, \mathbf{256}) + (\mathbf{1}, \mathbf{1}, \mathbf{120}) + (\mathbf{1}, \mathbf{1}, \overline{\mathbf{120}}) \right]_{spin\ 0} , \\
88 & : (\mathbf{64}, \mathbf{1}, \mathbf{1})_R + (\mathbf{28}, \mathbf{1}, \mathbf{1})_L + (\overline{\mathbf{28}}, \mathbf{1}, \mathbf{1})_L , \\
8'8' & : (\mathbf{1}, \mathbf{64}, \mathbf{1})_L + (\mathbf{1}, \mathbf{28}, \mathbf{1})_R + (\mathbf{1}, \overline{\mathbf{28}}, \mathbf{1})_R , \\
86, 8'6 & : (\mathbf{8}, \mathbf{1}, \mathbf{16})_L + (\mathbf{1}, \mathbf{8}, \mathbf{16})_R ,
\end{aligned} \tag{2.2}$$

where the massless 66 states are only scalars and the subscripts indicate the 6d chirality of the fermions. Notice that the D6 branes, parallel to the direction of supersymmetry breaking, feel its effects at tree level, while all massless sectors related to the D8 and D8' branes are supersymmetric. It should be appreciated that the 6d chirality of the fermion sectors related to the D8' branes (8'8' and 8'6 sectors) is flipped with respect to the supersymmetric case, while all fermions related to the D8 branes (88 and 86 sectors) have the same chirality as in the supersymmetric case. This model could also be regarded as a compactification of M-theory on $S^1 \times T^4/(Z_2 \times Z_2)$, where each $U(8)$ factor would originate from the breaking of an E_8 gauge group, while $x_9(\equiv X)$ would be interpreted as the eleventh dimension [1]. However, in addition to the gravitational sector, the x_9 -bulk contains the D6 brane sector with a gauge group $U(16)$. From the M-theory viewpoint, this (bulk) gauge group has a nonperturbative origin, related to the M5 brane wrapped around the compact space.

Consider now a similar Scherk-Schwarz deformation of the supersymmetric 4d $N = 1$ type I vacuum obtained from the type-IIB string compactified on $Z'_6 \equiv Z_2 \times Z_3$, where the orbifold action on the three complex internal planes is generated by

$$Z_2 = (1, -1, -1), \quad Z_3 = (e^{2\pi i/3}, e^{-2\pi i/3}, 1). \tag{2.3}$$

We can introduce a $(-1)^F$ deformation acting as a 2π rotation on one of the coordinates of the third complex plane and on one of the non-compact coordinates. This deformation is compatible with both orbifold operations and induces the spontaneous breaking of supersymmetry from $N = 1$ to $N = 0$. Since the Z_3 orbifold does not give rise to additional branes, the open string spectrum may be simply deduced by a Z_3 projection of the spectrum in eq. (2.2).

In order to gain a better understanding of the breaking pattern, we begin by recalling some features of the supersymmetric Z'_6 model. This contains 32 D9 branes and 32 D5 branes, that we can initially place at the origin [18]. The Z_2 orbifold inverts half of the Chan-Paton charges both for D9 and for D5 branes. Thus, for instance, the N charges associated to D9-branes are projected according to

$$n + \bar{n} \rightarrow i(n - \bar{n}) \quad . \quad (2.4)$$

Combining the Z_2 orbifold with the Z_3 projection

$$n \rightarrow n_0 + e^{\frac{2\pi i}{3}} n_1 + e^{-\frac{2\pi i}{3}} n_2 \quad , \quad (2.5)$$

then leads to a model with N=1 supersymmetry, a gauge group $[U(4) \times U(4) \times U(8)]_9 \times [U(4) \times U(4) \times U(8)]_5$, and chiral matter in the representations

$$\begin{aligned} 99 \text{ or } 55 \quad : & \quad (\mathbf{4}, \mathbf{4}, \mathbf{1}) + (\bar{\mathbf{4}}, \bar{\mathbf{4}}, \mathbf{1}) + (\bar{\mathbf{4}}, \mathbf{4}, \mathbf{1}) + (\mathbf{6}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \bar{\mathbf{6}}, \mathbf{1}) + \\ & \quad (\mathbf{1}, \mathbf{1}, \mathbf{28}) + (\mathbf{1}, \mathbf{1}, \bar{\mathbf{28}}) + (\mathbf{1}, \mathbf{4}, \mathbf{8}) + (\bar{\mathbf{4}}, \mathbf{1}, \bar{\mathbf{8}}) + (\mathbf{4}, \mathbf{1}, \bar{\mathbf{8}}) + (\mathbf{1}, \bar{\mathbf{4}}, \mathbf{8}) \quad , \\ 59 \quad : & \quad (\mathbf{1}, \mathbf{4}, \mathbf{1}; \mathbf{1}, \mathbf{4}, \mathbf{1}) + (\mathbf{4}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \mathbf{8}) + (\mathbf{1}, \mathbf{1}, \mathbf{8}; \mathbf{4}, \mathbf{1}, \mathbf{1}) + \\ & \quad (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{1}; \bar{\mathbf{4}}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \bar{\mathbf{4}}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \bar{\mathbf{8}}) + (\mathbf{1}, \mathbf{1}, \bar{\mathbf{8}}; \mathbf{1}, \bar{\mathbf{4}}, \mathbf{1}) \quad . \end{aligned} \quad (2.6)$$

In analogy with the 6d model, we can relate this spectrum to an M-theory compactification on $S^1 \times T^6 / (Z_2 \times Z'_6)$, turning on the same Wilson line breaking of the D9 gauge group $U(4) \times U(4) \times U(8)$ to $[U(2) \times U(2) \times U(4)]^2$ and then performing a T-duality to the type

I' description. The resulting open spectrum can be obtained by the Z_3 truncation (2.5) of the 6d spectrum (2.1), and thus contains the following $N = 1$ massless chiral matter multiplets (in a self-explanatory shorthand notation):

$$\begin{aligned}
66 & : (4, 4, 1) + (\bar{4}, \bar{4}, 1) + (\bar{4}, 4, 1) + (6, 1, 1) + (1, \bar{6}, 1) + \\
& (1, 1, 28) + (1, 1, \bar{28}) + (1, 4, 8) + (\bar{4}, 1, \bar{8}) + (4, 1, \bar{8}) + (1, \bar{4}, 8) , \\
88 \text{ or } 8'8' & : (\bar{2}, 2, 1) + (2, 2, 1) + (\bar{2}, \bar{2}, 1) + (2, 1, \bar{4}) + (\bar{2}, 1, \bar{4}) + \\
& (1, \bar{2}, 4) + (1, 2, 4) + (1, 1, 6) + (1, 1, \bar{6}) + (1, 1, 1) + (1, \bar{1}, 1) , \\
86 \text{ or } 8'6 & : (1, 2, 1; 1, 4, 1) + (1, 1, 4; 4, 1, 1) + (2, 1, 1; 1, 1, 8) + \\
& (\bar{2}, 1, 1; \bar{4}, 1, 1) + (1, 1, \bar{4}; 1, \bar{4}, 1) + (1, \bar{2}, 1; 1, 1, \bar{8}) ,
\end{aligned} \tag{2.7}$$

where by convention, all the 4d fermions displayed above are left-handed, and “barred” representations have $U(1)$ charges opposite to those of “unbarred” ones.

The open spectrum of the deformed, non-supersymmetric Z'_6 model, can similarly be obtained from the 6d spectrum of eq. (2.2) by the Z_3 truncation (2.5). In M-theory language, where $x_9(\equiv X)$ would be interpreted as the eleventh dimension, the identical D8 and D8' groups $[U(2) \times U(2) \times U(4)]$, of the Pati-Salam type, would result from the breaking of the two E_8 factors. However, as in the supersymmetric case, the x_9 -bulk contains both the gravitational sector and the D6 brane sector, with a gauge group $U(4) \times U(4) \times U(8)$. From the M-theory viewpoint, this gauge group is of non-perturbative origin, and results from the wrapping of the M5 brane around the compact space. Since this group lives in the bulk, it feels at tree-level the breaking of supersymmetry along the eleventh dimension. Therefore, in the D6 brane sector all fermions become massive at tree level and, in addition to the $U(4) \times U(4) \times U(8)$ gauge bosons, the massless spectrum contains only complex scalars in the representations

$$66 : (4, 4, 1) + (\bar{4}, \bar{4}, 1) + (\bar{4}, 4, 1) + (6, 1, 1) + (1, \bar{6}, 1) +$$

$$(\mathbf{1}, \mathbf{1}, \mathbf{28}) + (\mathbf{1}, \mathbf{1}, \overline{\mathbf{28}}) + (\mathbf{1}, \mathbf{4}, \mathbf{8}) + (\overline{\mathbf{4}}, \mathbf{1}, \overline{\mathbf{8}}) + (\mathbf{4}, \mathbf{1}, \overline{\mathbf{8}}) + (\mathbf{1}, \overline{\mathbf{4}}, \mathbf{8}) . \quad (2.8)$$

On the other hand, the massless spectra of the D8 and D8' branes, both transverse to x_9 , are supersymmetric. In addition to the $N = 1$ gauge multiplets, the D8 brane sector contains the massless chiral representations

$$\begin{aligned} 88 : & (\overline{\mathbf{2}}, \mathbf{2}, \mathbf{1}) + (\mathbf{2}, \mathbf{2}, \mathbf{1}) + (\overline{\mathbf{2}}, \overline{\mathbf{2}}, \mathbf{1}) + (\mathbf{2}, \mathbf{1}, \overline{\mathbf{4}}) + (\overline{\mathbf{2}}, \mathbf{1}, \overline{\mathbf{4}}) + \\ & (\mathbf{1}, \overline{\mathbf{2}}, \mathbf{4}) + (\mathbf{1}, \mathbf{2}, \mathbf{4}) + (\mathbf{1}, \mathbf{1}, \mathbf{6}) + (\mathbf{1}, \mathbf{1}, \overline{\mathbf{6}}) + (\mathbf{1}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \overline{\mathbf{1}}, \mathbf{1}) . \end{aligned} \quad (2.9)$$

The massless spectrum of the D8' gauge factor contains the corresponding conjugate representations, a phenomenon that can be traced to the chirality change in the parent 6d model. There are no 88' massless representations mixing the two “boundary” gauge factors.

The mixed 86 and 8'6 sectors, also supersymmetric at tree level, contain the massless representations

$$\begin{aligned} 86 : & (\mathbf{1}, \mathbf{2}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{4}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{4}; \mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{4}, \mathbf{1}, \mathbf{1}) + (\mathbf{2}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \mathbf{8}) + \\ & (\overline{\mathbf{2}}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \mathbf{1}; \overline{\mathbf{4}}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \overline{\mathbf{4}}; \mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \overline{\mathbf{4}}, \mathbf{1}) + (\mathbf{1}, \overline{\mathbf{2}}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \overline{\mathbf{8}}) , \\ 8'6 : & (\mathbf{1}, \mathbf{1}, \mathbf{1}; \overline{\mathbf{1}}, \overline{\mathbf{2}}, \overline{\mathbf{1}}; \overline{\mathbf{1}}, \overline{\mathbf{4}}, \overline{\mathbf{1}}) + (\mathbf{1}, \mathbf{1}, \mathbf{1}; \overline{\mathbf{1}}, \overline{\mathbf{1}}, \overline{\mathbf{4}}; \overline{\mathbf{4}}, \overline{\mathbf{1}}, \overline{\mathbf{1}}) + (\mathbf{1}, \mathbf{1}, \mathbf{1}; \overline{\mathbf{2}}, \overline{\mathbf{1}}, \overline{\mathbf{1}}; \overline{\mathbf{1}}, \overline{\mathbf{1}}, \overline{\mathbf{8}}) + \\ & (\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{2}, \overline{\mathbf{1}}, \overline{\mathbf{1}}; \mathbf{4}, \overline{\mathbf{1}}, \overline{\mathbf{1}}) + (\mathbf{1}, \mathbf{1}, \mathbf{1}; \overline{\mathbf{1}}, \overline{\mathbf{1}}, \mathbf{4}; \overline{\mathbf{1}}, \mathbf{4}, \overline{\mathbf{1}}) + (\mathbf{1}, \mathbf{1}, \mathbf{1}; \overline{\mathbf{1}}, \mathbf{2}, \overline{\mathbf{1}}; \overline{\mathbf{1}}, \overline{\mathbf{1}}, \mathbf{8}) , \end{aligned} \quad (2.10)$$

where the labels refer to the nine factors of the gauge group $[U(2) \times U(2) \times U(4)]_8 \times [U(2) \times U(2) \times U(4)]_{8'} \times [U(4) \times U(4) \times U(8)]_6$. Notice again the change of chirality (or complex conjugation) in the representations of the D8' gauge group compared to the supersymmetric case (2.7). Of course, one can interchange the roles of D9 and D5 branes in this model simply coupling (in type-I language) the supersymmetry breaking deformation to the momentum modes of X , rather than to the winding modes.

3. $N = 4 \rightarrow N = 2$ supersymmetry breaking in the closed sector

In [1] and in the previous Section we have considered a Scherk-Schwarz deformation along a compact coordinate X induced by the fermion parity $(-1)^F$. As we have seen, this breaks supersymmetry completely in the closed sector and in the open sectors that correspond to D-branes parallel to the coordinate X . In this Section we consider the effects of other (discrete) symmetries, such as ordinary R-symmetries, that correspond to global rotations in the 6d internal space. As we shall see, in general these deformations result in partial breakings of supersymmetry. For heterotic and type II strings, models of this type were first constructed by Kiritsis and Kounnas [11], who also showed their equivalent interpretation as freely-acting orbifolds [12]. Their results are our starting point for the construction of open descendants.

For pedagogical reasons, we begin our study from 5d compactifications. These provide the simplest examples of partial supersymmetry breaking from $N = 2$ to $N = 1$ in 5d, or equivalently, after an additional circle reduction to four dimensions, from $N = 4$ to $N = 2$. We thus consider the type IIB string compactified on a five-torus $T^4 \times S^1$ and couple the momenta or the windings along the S^1 coordinate x_5 to the charges of a discrete symmetry associated to a current J , that induce a simultaneous π rotation on the two complex internal planes $X_3 \equiv x_6 + ix_7$ and $X_4 \equiv x_8 + ix_9$ and on their world-sheet fermionic superpartners $\Psi_3 \equiv \psi_6 + i\psi_7$ and $\Psi_4 \equiv \psi_8 + i\psi_9$. Alternatively, under this rotation all four bosonic and fermionic coordinates of T^4 , x_6, \dots, x_9 and ψ_6, \dots, ψ_9 , change sign. Denoting by $|s_1 s_2 s_3 s_4\rangle$ the spacetime fermions, with $s_i (= \pm 1/2)$ the external (s_1, s_2) and internal (s_3, s_4) helicities with respect to the $SO(2)^4$ decomposition of the 10d $SO(8)$ little group, one can represent the action of the deformation on massless states as $\exp(i\pi \oint J) = \exp[i\pi(s_3 + s_4)]$. Obviously, this does not affect the massless gravitini with $s_3 = -s_4$, and therefore breaks only half of the supersymmetries. Taking into account the Ω projection for type I descendants, after an additional circle reduction to 4d this describes a partial breaking from $N = 4$ to $N = 2$.

With a momentum shift, in the zero-winding sector the SS deformation reduces to the field-theoretic boundary conditions for higher dimensional fields Φ :

$$\Phi(x_5 + 2\pi R) = U\Phi(x_5) \quad ; \quad U = e^{i\pi \oint J} \quad (U^2 = 1) . \quad (3.1)$$

In the rest of the closed spectrum, the SS deformation is then uniquely determined by modular invariance, and the torus partition function reads [11]

$$T = T_{N=4} + \sum'_{\tilde{M}, N} \left| \sum_{a, b=0, 1/2} (-1)^{2(a+b+ab)} \frac{\theta^2 \left(\begin{smallmatrix} a \\ b \end{smallmatrix} \right) \theta \left(\begin{smallmatrix} a - N/2 \\ b + \tilde{M}/2 \end{smallmatrix} \right) \theta \left(\begin{smallmatrix} a + N/2 \\ b - \tilde{M}/2 \end{smallmatrix} \right)}{\eta^6 \theta \left(\begin{smallmatrix} 1/2 - N/2 \\ 1/2 + \tilde{M}/2 \end{smallmatrix} \right) \theta \left(\begin{smallmatrix} 1/2 + N/2 \\ 1/2 - \tilde{M}/2 \end{smallmatrix} \right)} \right|^2 Z_{\tilde{M}, N} , \quad (3.2)$$

where $T_{N=4}$ is the supersymmetric type IIB torus partition function on $T^4 \times S^1$, and $Z_{\tilde{M}, N}$, with \tilde{M} the Poisson resummed momentum, is the S^1 lattice sum for the circle coordinate $X(\equiv x_5)$ coupled to the J -charges. In addition, the θ 's are Jacobi theta-functions depending on the modular parameter τ of the world-sheet torus and the “primed” sum over (\tilde{M}, N) in the second term in (3.2) does not contain the *(even, even)* part. After the redefinitions $N = 2n + k$ and $\tilde{M} = 2\tilde{m} + l$ with $l, k = 0, 1$, one finds (omitting the contribution of the transverse bosons)

$$T = \frac{1}{2} \{ |Q_O + Q_V|^2 \Lambda_{4,4} Z_{2\tilde{m}, 2n} + |Q_O - Q_V|^2 |I_4 I_4 - V_4 V_4|_B^2 Z_{2\tilde{m}+1, 2n} \\ + |Q_S + Q_C|^2 |Q_S + Q_C|_B^2 Z_{2\tilde{m}, 2n+1} + |Q_S - Q_C|^2 |Q_S - Q_C|_B^2 Z_{2\tilde{m}+1, 2n+1} \} \quad , \quad (3.3)$$

where $\Lambda_{4,4}$ is the lattice sum for T^4 and, following [5], we have introduced the convenient notation

$$Q_O = V_4 I_4 - C_4 C_4 , \quad Q_V = I_4 V_4 - S_4 S_4 , \\ Q_S = I_4 C_4 - S_4 I_4 , \quad Q_C = V_4 S_4 - C_4 V_4 . \quad (3.4)$$

I_n, V_n, S_n, C_n denote the standard $SO(2n)$ level-one characters, corresponding to the conjugacy classes of the identity, the vector and the two chiral spinors, while the subscript B

refers to the compact bosonic modes, fermionized according to

$$\begin{aligned} 4\frac{\eta^2}{\theta_2^2} &= \frac{\theta_3^2\theta_4^2}{\eta^4} = (I_4I_4 - V_4V_4)_B, \quad 4\frac{\eta^2}{\theta_4^2} = \frac{\theta_2^2\theta_3^2}{\eta^4} = (Q_S + Q_C)_B, \\ 4\frac{\eta^2}{\theta_3^2} &= \frac{\theta_2^2\theta_4^2}{\eta^4} = (Q_S - Q_C)_B. \end{aligned} \quad (3.5)$$

After a Poisson resummation in \tilde{m} , one can recast eq. (3.3) in the form⁴

$$\begin{aligned} T &= \frac{1}{2} \{ |Q_O + Q_V|^2 \Lambda_{4,4} (Z_{m,2n} + Z_{m+\frac{1}{2},2n}) + |Q_O - Q_V|^2 |I_4I_4 - V_4V_4|_B^2 (Z_{m,2n} - Z_{m+\frac{1}{2},2n}) \\ &\quad + |Q_S + Q_C|^2 |Q_S + Q_C|_B^2 (Z_{m,2n+1} + Z_{m+\frac{1}{2},2n+1}) \\ &\quad + |Q_S - Q_C|^2 |Q_S - Q_C|_B^2 (Z_{m,2n+1} - Z_{m+\frac{1}{2},2n+1}) \} . \end{aligned} \quad (3.6)$$

As discussed in [11], after halving the radius R to $R/2$ this can be reinterpreted as the partition function of a freely-acting orbifold. As can be seen from eq. (3.1), in this case the relevant orbifold is $IIB/(-1)^m\mathcal{I}$, where $(-1)^m$ denotes the order-two shift $x_5 \rightarrow x_5 + \pi R$ and \mathcal{I} denotes the inversion of the four internal coordinates, $\mathcal{I}x_{6,\dots,9} = -x_{6,\dots,9}$. Our notation is particularly convenient, since \mathcal{I} has a very simple action on the basis (3.4): it inverts Q_V and Q_C and leaves Q_O and Q_S unaffected. The resulting form of the torus amplitude,

$$\begin{aligned} T &= \frac{1}{2} \{ |Q_O + Q_V|^2 \Lambda_{4,4} Z_{m,n} + |Q_O - Q_V|^2 |I_4I_4 - V_4V_4|_B^2 (-1)^m Z_{m,n} \\ &\quad + |Q_S + Q_C|^2 |Q_S + Q_C|_B^2 Z_{m,n+\frac{1}{2}} + |Q_S - Q_C|^2 |Q_S - Q_C|_B^2 (-1)^m Z_{m,n+\frac{1}{2}} \} , \end{aligned} \quad (3.7)$$

will be the starting point for the construction of the Scherk-Schwarz breaking model in the next Section.

Alternatively, one can couple the J -charges to the winding modes of X . The corresponding torus amplitude, simply obtained interchanging m and n in (3.6) and (3.7), will be our starting point for the construction of the M-theory breaking model in Section 5.

⁴From now on we omit the contribution of the transverse space-time bosons.

4. $N = 4 \rightarrow N = 2$ Scherk-Schwarz breaking

Using (3.7) and the standard methods of [3, 4, 5], one can obtain the direct-channel Klein bottle amplitude

$$K = \frac{1}{4}(Q_O + Q_V)[(P+W)Z_{2m} + (P-W)Z_{2m+1}] = \frac{1}{4}(Q_O + Q_V)[PZ_m + W(-1)^m Z_m] , \quad (4.1)$$

where P and W denote the momentum and winding sums for T^4 and Z_m denotes the momentum sum along X . Notice that, in the Klein bottle, the orbifold projection acts effectively as $(-1)^m$. In the three relevant amplitudes (Klein bottle, annulus and Möbius-strip), the relations between the direct-channel parameter t and the transverse-channel parameter l are

$$\begin{aligned} K & : \tau = 2it_K , \quad l = \frac{1}{2t_K} , \\ A & : \tau = \frac{it_A}{2} , \quad l = \frac{2}{t_A} , \\ M & : \tau = \frac{it_M}{2} + \frac{1}{2} , \quad l = \frac{1}{2t_M} . \end{aligned} \quad (4.2)$$

The direct and transverse Klein-bottle and annulus amplitudes are thus related by an S transformation while, in a real basis of “hatted” characters, for the Möbius strip the corresponding transformation is effected by the sequence $P = T^{1/2}ST^2ST^{1/2}$ [5]. Here, as usual, S denotes the modular inversion, $\tau \rightarrow -1/\tau$, while T denotes the modular shift $\tau \rightarrow \tau + 1$.

The transverse-channel Klein bottle amplitude is then

$$\tilde{K} = \frac{2^5}{4}R(Q_O + Q_V)[v_4 W^e \tilde{Z}_{2n} + \frac{P^e}{v_4} \tilde{Z}_{2n+1}] , \quad (4.3)$$

with P^e (W^e) the even momentum (winding) sums for the T^4 lattice and v_4 the T^4 volume. Since the only massless tadpole contribution in \tilde{K} is proportional to v_4 , one can anticipate that the open spectrum will contain only D9 branes.

From (3.7) one can also read the transverse-channel annulus amplitude. Introducing the minimum number of independent reflection coefficients $\alpha, \beta, \gamma, \delta$,

$$\begin{aligned} \tilde{A} = & \frac{2^{-5}}{4} R \{ [(Q_O + Q_V)(\alpha^2 v_4 W + \beta^2 \frac{P}{v_4}) + 2\alpha\beta(Q_O - Q_V)(I_4 I_4 - V_4 V_4)_B] \tilde{Z}_n \\ & + [(\gamma^2 + \delta^2)(Q_S + Q_C)(Q_S + Q_C)_B + 2\gamma\delta(Q_S - Q_C)(Q_S - Q_C)_B] \tilde{Z}_{n+\frac{1}{2}} \} . \end{aligned} \quad (4.4)$$

In the direct-channel amplitude corresponding to (4.4), α and γ are related to Neumann (N) charges, while β and δ are related to Dirichlet (D) charges. Thus the cross terms, proportional to $\alpha\beta$ and $\gamma\delta$, determine the ND sector of the model.

A basic test of the consistency of these amplitudes is obtained from the terms at the origin of the T^4 lattice, whose reflection coefficients are to arrange in perfect squares. In the following three equations, for notational simplicity we omit the B subscript for the contributions of the internal bosons. Then,

$$\tilde{K}_0 = \frac{2^5}{4} R [(Q_O I_4 I_4 + Q_V V_4 V_4) + (Q_O V_4 V_4 + Q_V I_4 I_4)] (v_4 \tilde{Z}_{2n} + \frac{1}{v_4} \tilde{Z}_{2n+1}) \quad (4.5)$$

and

$$\tilde{A}_0 = \frac{2^{-5}}{4} R [(Q_O I_4 I_4 + Q_V V_4 V_4) (\sqrt{v_4} \alpha + \frac{\beta}{\sqrt{v_4}})^2 \tilde{Z}_n + (Q_O V_4 V_4 + Q_V I_4 I_4) (\sqrt{v_4} \alpha - \frac{\beta}{\sqrt{v_4}})^2 \tilde{Z}_n] . \quad (4.6)$$

The geometric mean of the characters common to \tilde{K}_0 and \tilde{A}_0 determines the transverse-channel Möbius amplitude at the origin of the T^4 lattice,

$$\begin{aligned} \tilde{M}_0 = & -\frac{R}{2} \{ [\sqrt{v_4} (\sqrt{v_4} \alpha + \frac{\beta}{\sqrt{v_4}}) \tilde{Z}_{2n} + \frac{1}{\sqrt{v_4}} (\sqrt{v_4} \alpha + \frac{\beta}{\sqrt{v_4}}) \tilde{Z}_{2n+1}] (\hat{Q}_O \hat{I}_4 \hat{I}_4 + \hat{Q}_V \hat{V}_4 \hat{V}_4) \\ & + [\sqrt{v_4} (\sqrt{v_4} \alpha - \frac{\beta}{\sqrt{v_4}}) \tilde{Z}_{2n} - \frac{1}{\sqrt{v_4}} (\sqrt{v_4} \alpha - \frac{\beta}{\sqrt{v_4}}) \tilde{Z}_{2n+1}] (\hat{Q}_O \hat{V}_4 \hat{V}_4 + \hat{Q}_V \hat{I}_4 \hat{I}_4) \} , \end{aligned} \quad (4.7)$$

where in the “hatted” characters, defined as in [1], the argument is shifted by $1/2$, consistently with eq. (4.2) [4, 5]. From the contributions at the origin, one can deduce the full transverse-channel Möbius amplitude:

$$\tilde{M} = -\frac{R}{2} [(\hat{Q}_O + \hat{Q}_V)(\alpha v_4 W^e \tilde{Z}_{2n} + \beta \frac{P^e}{v_4} \tilde{Z}_{2n+1}) + (\hat{Q}_O - \hat{Q}_V)(\hat{I}_4 \hat{I}_4 - \hat{V}_4 \hat{V}_4)_B (\beta \tilde{Z}_{2n} + \alpha \tilde{Z}_{2n+1})] . \quad (4.8)$$

Combining eqs. (4.3), (4.4) and (4.8), one can then derive the tadpole conditions for α and β ,

$$\alpha = 32 \quad , \quad \beta = 0 \quad . \quad (4.9)$$

Thus δ , the orbifold breaking of β , must also vanish, and one is led to the parametrization

$$\alpha = n_1 + n_2 = 32 \quad , \quad \gamma = n_1 - n_2 \quad , \quad (4.10)$$

in terms of which, after doubling the radius to return to the original Scherk-Schwarz basis (3.6), the open string amplitudes are⁵

$$\begin{aligned} A &= \frac{(n_1 + n_2)^2}{4} (Q_O + Q_V) P Z_m + \frac{(n_1 - n_2)^2}{4} (Q_O - Q_V) (I_4 I_4 - V_4 V_4)_B (-1)^m Z_m , \\ M &= -\frac{(n_1 + n_2)}{4} \{ (\hat{Q}_O + \hat{Q}_V) P Z_m + (\hat{Q}_O - \hat{Q}_V) (\hat{I}_4 \hat{I}_4 - \hat{V}_4 \hat{V}_4)_B (-1)^m Z_m \} . \end{aligned} \quad (4.11)$$

The resulting model has $N = 2$ supersymmetry in 4d and a gauge group $SO(n_1) \times SO(n_2)$ originating from D9 branes. This is manifestly a Scherk-Schwarz deformation of the $N = 4$ supersymmetric 6d type I model with a Wilson line that breaks the $SO(32)$ gauge group of the D9 branes to $SO(n_1) \times SO(n_2)$, in analogy with the first model described in [1]. For instance, at the origin of the T^4 lattice, all integer $(2m)$ momentum levels contain the adjoint vector multiplets and one hypermultiplet in the representation $(\mathbf{n}_1, \mathbf{n}_2)$, while all half-integer momentum levels contain one vector multiplet in the $(\mathbf{n}_1, \mathbf{n}_2)$ representation and hypermultiplets in the adjoint representations $(\mathbf{n}_1(\mathbf{n}_1 - \mathbf{1})/2, \mathbf{1}) + (\mathbf{1}, \mathbf{n}_2(\mathbf{n}_2 - \mathbf{1})/2)$. The particular choice $n_2 = 0$ corresponds to a trivial Wilson line, and thus to the unbroken $SO(32)$ D9 gauge group. As expected, $N = 4$ supersymmetry is recovered in the $R \rightarrow \infty$ limit, without any additional constraints on the models.

⁵With our convention $\alpha' = 2$, in the direct channel annulus and Möbius amplitudes Z_{2m} describes integer momentum levels, while Z_{2m+1} describes half-integer momentum levels.

5. $N = 4 \rightarrow N = 2$ M-theory breaking

In this case the starting point is

$$T = \frac{1}{2} \{ |Q_O + Q_V|^2 \Lambda_{4,4} Z_{m,n} + |Q_O - Q_V|^2 |I_4 I_4 - V_4 V_4|_B^2 (-1)^n Z_{m,n} \\ + |Q_S + Q_C|^2 |Q_S + Q_C|_B^2 Z_{m+\frac{1}{2},n} + |Q_S - Q_C|^2 |Q_S - Q_C|_B^2 (-1)^n Z_{m+\frac{1}{2},n} \} , \quad (5.1)$$

with the understanding that the Scherk-Schwarz (M-theory) basis is obtained after halving the radius. Following the same steps as in the first model, one finds the direct channel Klein-bottle amplitude

$$K = \frac{1}{4} [(Q_O + Q_V)(P + W)Z_m + 2(Q_S + Q_C)(Q_S + Q_C)_B Z_{m+\frac{1}{2}}] \quad (5.2)$$

and, after an S-transformation, the transverse-channel amplitude

$$\tilde{K} = \frac{2^5}{4} R [(Q_O + Q_V)(v_4 W^e + \frac{P^e}{v_4}) \tilde{Z}_{2n} + 2(Q_O - Q_V)(I_4 I_4 - V_4 V_4)_B (-1)^n \tilde{Z}_{2n}] , \quad (5.3)$$

where the superscript e identifies the even terms in the T^4 lattice sums. From the massless $1/v_4$ tadpole in \tilde{K} , one may anticipate the existence of one set of D5 branes, in addition to the D9 branes related to the tadpole contribution proportional to v_4 .

Starting from (5.1), one can then find the transverse annulus amplitude. Introducing the minimum required number of Chan-Paton charges $(\alpha_1, \alpha_2, \beta_1, \beta_2)$ in a convenient way,

$$\tilde{A} = \frac{2^5}{4} R \{ [(Q_O + Q_V)(\alpha_1^2 W v_4 + \beta_1^2 \frac{P}{v_4}) + 2\alpha_1 \beta_1 (Q_O - Q_V)(I_4 I_4 - V_4 V_4)_B] \tilde{Z}_{4n} \\ + [(Q_O + Q_V)(\alpha_2^2 W v_4 + \beta_2^2 \frac{P}{v_4}) - 2\alpha_2 \beta_2 (Q_O - Q_V)(I_4 I_4 - V_4 V_4)_B] \tilde{Z}_{4n+2} \} . \quad (5.4)$$

As usual, the origin of the T^4 lattice allows one to identify the Möbius amplitude. In the following three equations, we omit again the B subscript for the contributions of the internal bosons. The geometric mean of the coefficients of the characters common to

$$\tilde{K}_0 = \frac{2^5}{4} R \{ [(Q_O I_4 I_4 + Q_V V_4 V_4)(\sqrt{v_4} + \frac{1}{\sqrt{v_4}})^2 + (Q_O V_4 V_4 + Q_V I_4 I_4)(\sqrt{v_4} - \frac{1}{\sqrt{v_4}})^2] \tilde{Z}_{4n} \\ + [(Q_O I_4 I_4 + Q_V V_4 V_4)(\sqrt{v_4} - \frac{1}{\sqrt{v_4}})^2 + (Q_O V_4 V_4 + Q_V I_4 I_4)(\sqrt{v_4} + \frac{1}{\sqrt{v_4}})^2] \tilde{Z}_{4n+2} \} \quad (5.5)$$

and

$$\begin{aligned}\tilde{A}_0 = & \frac{2^{-5}}{4} R \{ [(Q_O I_4 I_4 + Q_V V_4 V_4) (\alpha_1 \sqrt{v_4} + \frac{\beta_1}{\sqrt{v_4}})^2 + (Q_O V_4 V_4 + Q_V I_4 I_4) (\alpha_1 \sqrt{v_4} - \frac{\beta_1}{\sqrt{v_4}})^2] \tilde{Z}_{4n} \\ & + [(Q_O I_4 I_4 + Q_V V_4 V_4) (\alpha_2 \sqrt{v_4} - \frac{\beta_2}{\sqrt{v_4}})^2 + (Q_O V_4 V_4 + Q_V I_4 I_4) (\alpha_2 \sqrt{v_4} + \frac{\beta_2}{\sqrt{v_4}})^2] \tilde{Z}_{4n+2} \} \quad (5.6)\end{aligned}$$

determines in the transverse Möbius amplitude the contributions at the origin of the T^4 lattice:

$$\begin{aligned}\tilde{M}_0 = & -\frac{R}{2} \{ (\hat{Q}_O \hat{I}_4 \hat{I}_4 + \hat{Q}_V \hat{V}_4 \hat{V}_4) [(\alpha_1 v_4 + \frac{\beta_1}{v_4} + \alpha_1 + \beta_1) \tilde{Z}_{4n} + (\alpha_2 v_4 + \frac{\beta_2}{v_4} - \beta_2 - \alpha_2) \tilde{Z}_{4n+2}] \\ & + (\hat{Q}_O \hat{V}_4 \hat{V}_4 + \hat{Q}_V \hat{I}_4 \hat{I}_4) [(\alpha_1 v_4 + \frac{\beta_1}{v_4} - \alpha_1 - \beta_1) \tilde{Z}_{4n} + (\alpha_2 v_4 + \frac{\beta_2}{v_4} + \alpha_2 + \beta_2) \tilde{Z}_{4n+2}] \} . \quad (5.7)\end{aligned}$$

These suffice to reconstruct the full transverse-channel Möbius amplitude, that reads

$$\begin{aligned}\tilde{M} = & -\frac{R}{2} \{ [(\hat{Q}_O + \hat{Q}_V) (\alpha_1 W^e v_4 + \beta_1 \frac{P^e}{v_4}) + (\alpha_1 + \beta_1) (\hat{Q}_O - \hat{Q}_V) (\hat{I}_4 \hat{I}_4 - \hat{V}_4 \hat{V}_4)_B] \tilde{Z}_{4n} \\ & + [(\hat{Q}_O + \hat{Q}_V) (\alpha_2 W^e v_4 + \beta_2 \frac{P^e}{v_4}) - (\alpha_2 + \beta_2) (\hat{Q}_O - \hat{Q}_V) (\hat{I}_4 \hat{I}_4 - \hat{V}_4 \hat{V}_4)_B] \tilde{Z}_{4n+2} \} . \quad (5.8)\end{aligned}$$

Eqs. (5.3), (5.4) and (5.8) then lead to the tadpole conditions

$$\alpha_1 = 32 \quad , \quad \beta_1 = 32 \quad , \quad (5.9)$$

while the corresponding charge parametrization is

$$\alpha_1 = 2(n_1 + n_2) \quad , \quad \alpha_2 = 2(n_1 - n_2) \quad , \quad \beta_1 = 2(d_1 + d_2) \quad , \quad \beta_2 = 2(d_1 - d_2) \quad . \quad (5.10)$$

In order to display the spectrum, it is convenient to halve the radius, reverting to the Scherk-Schwarz basis. With the charge assignments (5.10), the final result for the direct-channel annulus amplitude is

$$\begin{aligned}A = & \frac{1}{2} \{ [(n_1^2 + n_2^2) P + (d_1^2 + d_2^2) W] (Q_O + Q_V) + 2(n_1 d_2 + n_2 d_1) (Q_S + Q_C) (\frac{Q_S + Q_C}{4})_B \} Z_{2m} \\ & + \{ (n_1 n_2 P + d_1 d_2 W) (Q_O + Q_V) + (n_1 d_1 + n_2 d_2) (Q_S + Q_C) (\frac{Q_S + Q_C}{4})_B \} Z_{2m+1} , \quad (5.11)\end{aligned}$$

while, after a P transformation, the direct-channel Möbius amplitude is

$$M = -\frac{1}{2}\{(n_1P + d_1W)(\hat{Q}_O + \hat{Q}_V) - (n_2 + d_2)(\hat{Q}_O - \hat{Q}_V)(\hat{I}_4\hat{I}_4 - \hat{V}_4\hat{V}_4)_B\}Z_{4m} \\ -\frac{1}{2}\{(n_2P + d_2W)(\hat{Q}_O + \hat{Q}_V) - (n_1 + d_1)(\hat{Q}_O - \hat{Q}_V)(\hat{I}_4\hat{I}_4 - \hat{V}_4\hat{V}_4)_B\}Z_{4m+2}. \quad (5.12)$$

It is instructive to display the massless open spectrum of this model, encoded in

$$A = \frac{n_1^2 + n_2^2 + d_1^2 + d_2^2}{2}Q_O + \frac{n_1^2 + n_2^2 + d_1^2 + d_2^2}{2}Q_V + (n_1d_2 + n_2d_1)(Q_S + Q_C) \\ M = -\frac{n_1 - n_2 + d_1 - d_2}{2}\hat{Q}_O - \frac{n_1 + n_2 + d_1 + d_2}{2}\hat{Q}_V. \quad (5.13)$$

It should be appreciated that the gauge group, $SO(n_1) \times Sp(n_2) \times SO(d_1) \times Sp(d_2)$, with $n_1 + n_2 = 16$, $d_1 + d_2 = 16$, has a reduced rank with respect to standard orbifold models. Moreover, the first two gauge factors originate from 16 D9 branes, while the last two originate from 16 D5 branes⁶.

There are a number of interesting subtleties in this model. First of all, as we have seen, the gauge group has a reduced rank. This phenomenon, familiar in toroidal models [19], where it is induced by quantized expectation values of the NS-NS antisymmetric tensor, here draws its origin from the asymmetric nature of the winding shifts, that indeed allows this construction only in the Z_2 case. Moreover, the open spectrum itself is rather peculiar, since away from the origin of the T^4 lattice only some of the states are symmetrized in a conventional fashion. This is the case, for instance, for the term proportional to $n_1^2PZ_{4m}$, while the term proportional to $n_1^2PZ_{4m+2}$ does not have a corresponding Möbius contribution. This unfamiliar projection is actually consistent, since one more subtlety helps to settle matters. Namely, the very normalization of the annulus is unconventional, as can be seen, for instance, comparing eqs. (5.11) and (4.11). In toroidal models, the lattice contributes an operator of the type $\exp(ip_Lx_L + ip_Rx_R)$ for each lattice point. The conventional Z_2 orbifold projection effectively halves the number of available lattice operators,

⁶After the T-duality to the type I' picture, these become D8 and D4 branes, respectively.

since the corresponding *cos* and *sin* terms are assigned opposite eigenvalues. Therefore, in terms of the orbifold counting, away from the origin the annulus describes effectively two copies of the minimal spectrum. The Möbius projection on these two copies then realizes the two cases found, for instance, in WZW models with an extended algebra [20]: the terms without a Möbius contribution describe a pair of sectors, one with symmetric and one with antisymmetric Chan-Paton matrices, while the terms with a Möbius contribution describe a pair of sectors, both with (anti)symmetric Chan-Paton matrices. Finally, the full $N = 4$ supersymmetry should be recovered in the $R \rightarrow 0$ limit, but in this limit all massive winding modes clearly become massless and give the additional tadpole conditions $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, or equivalently $n_2 = d_2 = 0$, in (5.12) and (5.13) [1, 13]. These leave only the $N=4$ ($Q_O + Q_V$) character at the massless level, and determine the unique gauge group $SO(16) \times SO(16)$, expected by M-theory duality arguments similar to those presented in [1]⁷.

Letting $n_2 = d_2 = 0$, one can then see from (5.11) that for the $(\mathbf{16}, \mathbf{16})$ mixed representation, corresponding to the term $n_1 d_1 (Q_S + Q_C)(Q_S + Q_C)_B Z_{2m+1}$, the momentum sum is shifted by $1/2R$. This Wilson line has a counterpart in the T-dual, type I' model, that naturally associates the two $SO(16)$ gauge groups to the two different Horava-Witten walls. In the M-theory picture related to this type-I' setting, partial supersymmetry breaking would then be induced by an R-parity symmetry coupled to the eleventh dimension ($x_{11} \equiv x_5$). Still, by duality arguments one would naively expect a Horava-Witten type picture with two D8 branes, each containing an $SO(16)$ gauge group. In our case, however, the D8 branes on one of the walls are actually wrapped around the T^4 torus and have effectively turned into D4 branes (or, before T-duality, into the D5 branes that we have found explicitly).

Let us finally mention that in 4d reductions, where the extra circle S^1 is replaced by

⁷Notice that, in the M-theory model of [1], the gauge group $SO(16) \times SO(16)$ is also singled out if one requires that these small-radius singularities be absent.

T^2 , there are in general several directions along T^2 that can accommodate the shift, but in the corresponding models the main physical results remain unchanged.

6. $N = 4 \rightarrow N = 1$ Scherk-Schwarz breaking

One can also break $N = 4$ to $N = 1$ deforming the compactification lattice along a pair of different directions. The corresponding operations, however, must be compatible with the group structure, and the resulting models are rather involved. Here, we would like to describe a simpler model that is actually based on a $(T^2)^3$ compactification with shifts along three directions [11]. In this case one has the three group operations $g = (\delta_1, -1, -1\delta_3)$, $h = (-1, -1\delta_2, \delta_3)$, $f = (-1\delta_1, \delta_2, -1)$ with a $Z_2 \times Z_2$ orbifold structure, where δ_i indicates an order-two momentum shift in the compact torus T_i^2 .

The amplitudes involve the 16 characters [21]

$$\begin{aligned}
\tau_{oo} &= V_2 I_2 I_2 I_2 + I_2 V_2 V_2 V_2 - S_2 S_2 S_2 S_2 - C_2 C_2 C_2 C_2 , \\
\tau_{og} &= I_2 V_2 I_2 I_2 + V_2 I_2 V_2 V_2 - C_2 C_2 S_2 S_2 - S_2 S_2 C_2 C_2 , \\
\tau_{oh} &= I_2 I_2 I_2 V_2 + V_2 V_2 V_2 I_2 - C_2 S_2 S_2 C_2 - S_2 C_2 C_2 S_2 , \\
\tau_{of} &= I_2 I_2 V_2 I_2 + V_2 V_2 I_2 V_2 - C_2 S_2 C_2 S_2 - S_2 C_2 S_2 C_2 , \\
\tau_{go} &= V_2 I_2 S_2 C_2 + I_2 V_2 C_2 S_2 - S_2 S_2 V_2 I_2 - C_2 C_2 I_2 V_2 , \\
\tau_{gg} &= I_2 V_2 S_2 C_2 + V_2 I_2 C_2 S_2 - S_2 S_2 I_2 V_2 - C_2 C_2 V_2 I_2 , \\
\tau_{gh} &= I_2 I_2 S_2 S_2 + V_2 V_2 C_2 C_2 - C_2 S_2 V_2 V_2 - S_2 C_2 I_2 I_2 , \\
\tau_{gf} &= I_2 I_2 C_2 C_2 + V_2 V_2 S_2 S_2 - S_2 C_2 V_2 V_2 - C_2 S_2 I_2 I_2 , \\
\tau_{ho} &= V_2 S_2 C_2 I_2 + I_2 C_2 S_2 V_2 - C_2 I_2 V_2 C_2 - S_2 V_2 I_2 S_2 , \\
\tau_{hg} &= I_2 C_2 C_2 I_2 + V_2 S_2 S_2 V_2 - C_2 I_2 I_2 S_2 - S_2 V_2 V_2 C_2 ,
\end{aligned}$$

$$\begin{aligned}
\tau_{hh} &= I_2 S_2 C_2 V_2 + V_2 C_2 S_2 I_2 - S_2 I_2 V_2 S_2 - C_2 V_2 I_2 C_2 , \\
\tau_{hf} &= I_2 S_2 S_2 I_2 + V_2 C_2 C_2 V_2 - C_2 V_2 V_2 S_2 - S_2 I_2 I_2 C_2 , \\
\tau_{fo} &= V_2 S_2 I_2 C_2 + I_2 C_2 V_2 S_2 - S_2 V_2 S_2 I_2 - C_2 I_2 C_2 V_2 , \\
\tau_{fg} &= I_2 C_2 I_2 C_2 + V_2 S_2 V_2 S_2 - C_2 I_2 S_2 I_2 - S_2 V_2 C_2 V_2 , \\
\tau_{fh} &= I_2 S_2 I_2 S_2 + V_2 C_2 V_2 C_2 - C_2 V_2 S_2 V_2 - S_2 I_2 C_2 I_2 , \\
\tau_{ff} &= I_2 S_2 V_2 C_2 + V_2 C_2 I_2 S_2 - C_2 V_2 C_2 I_2 - S_2 I_2 S_2 V_2 ,
\end{aligned} \tag{6.1}$$

and the torus contribution may be obtained from eq. (3.7), after a further Z_2 orbifold projection accompanied by shifts in the three relevant momenta (m_1, m_2, m_3) . The end result is

$$\begin{aligned}
T = \frac{1}{4} \{ & |\tau_{oo} + \tau_{og} + \tau_{oh} + \tau_{of}|^2 \Lambda_1 \Lambda_2 \Lambda_3 + |\tau_{oo} + \tau_{og} - \tau_{oh} - \tau_{of}|^2 (-1)^{m_1} \Lambda_1 \left| \frac{4\eta^2}{\theta_2^2} \right|^2 \\
& + |\tau_{oo} - \tau_{og} + \tau_{oh} - \tau_{of}|^2 (-1)^{m_3} \Lambda_3 \left| \frac{4\eta^2}{\theta_2^2} \right|^2 + |\tau_{oo} - \tau_{og} - \tau_{oh} + \tau_{of}|^2 (-1)^{m_2} \Lambda_2 \left| \frac{4\eta^2}{\theta_2^2} \right|^2 \\
& + |\tau_{go} + \tau_{gg} + \tau_{gh} + \tau_{gf}|^2 \Lambda_1^{n_1+\frac{1}{2}} \left| \frac{4\eta^2}{\theta_4^2} \right|^2 + |\tau_{go} + \tau_{gg} - \tau_{gh} - \tau_{gf}|^2 (-1)^{m_1} \Lambda_1^{n_1+\frac{1}{2}} \left| \frac{4\eta^2}{\theta_3^2} \right|^2 \\
& + |\tau_{ho} + \tau_{hg} + \tau_{hh} + \tau_{hf}|^2 \Lambda_3^{n_3+\frac{1}{2}} \left| \frac{4\eta^2}{\theta_4^2} \right|^2 + |\tau_{ho} - \tau_{hg} + \tau_{hh} - \tau_{hf}|^2 (-1)^{m_3} \Lambda_3^{n_3+\frac{1}{2}} \left| \frac{4\eta^2}{\theta_3^2} \right|^2 \\
& + |\tau_{fo} + \tau_{fg} + \tau_{fh} + \tau_{ff}|^2 \Lambda_2^{n_2+\frac{1}{2}} \left| 4 \frac{\eta^2}{\theta_4^2} \right|^2 + |\tau_{fo} - \tau_{fg} - \tau_{fh} + \tau_{ff}|^2 (-1)^{m_2} \Lambda_2^{n_2+\frac{1}{2}} \left| 4 \frac{\eta^2}{\theta_3^2} \right|^2 \} , \tag{6.2}
\end{aligned}$$

where Λ_i , ($i = 1, 2, 3$) are the lattice sums for the three compact tori and, for instance, the shorthand notation $(-1)^{m_i} \Lambda_i^{n_i+1/2}$ indicates a sum with the insertion of $(-1)^{m_i}$ along one of the two momenta of T_i^2 , and with the corresponding windings shifted by $1/2$ unit. Notice that the torus (6.2) has indeed the structure of a $Z_2 \times Z_2$ orbifold, but all lattice independent terms are absent, since the (winding) shifts in the twisted sector eliminate all terms at the origin of the lattice for the three tori. Equivalently, from a geometric viewpoint, this reflects the nontrivial action of the projection on the would-be fixed points.

The Klein bottle amplitude in the direct channel is then

$$K = \frac{1}{8}(\tau_{oo} + \tau_{og} + \tau_{oh} + \tau_{of})\{P_1 P_2 P_3 + (-1)^{m_1} P_1 W_2 W_3 + W_1 (-1)^{m_2} P_2 W_3 + W_1 W_2 (-1)^{m_3} P_3\}, \quad (6.3)$$

where P_1, P_2, P_3 (W_1, W_2, W_3) denote the momentum (winding) sums for the three tori. By an S transformation, one can obtain the transverse-channel amplitude

$$\tilde{K} = \frac{2^5}{8}(\tau_{oo} + \tau_{og} + \tau_{oh} + \tau_{of})\left\{v_1 v_2 v_3 W_1^e W_2^e W_3^e + \frac{v_1}{v_2 v_3} W_1^o P_2^e P_3^e + \frac{v_2}{v_1 v_3} P_1^e W_2^o P_3^e + \frac{v_3}{v_1 v_2} P_1^e P_2^e W_3^o\right\}, \quad (6.4)$$

where v_1, v_2, v_3 are the volumes of the three compact tori and the superscripts e (o) in (6.4) indicate the even (odd) parts of the corresponding sums. Since the only massless tadpole in \tilde{K} is proportional to $v_1 v_2 v_3$, the open string spectrum will contain only D9 branes.

From the torus amplitude, introducing the minimum required number of Chan-Paton charges I_N, g_N, h_N, f_N , one can deduce the annulus amplitude in the transverse channel:

$$\begin{aligned} \tilde{A} = \frac{2^{-5}}{8} \{ & I_N^2 (\tau_{oo} + \tau_{og} + \tau_{oh} + \tau_{of}) v_1 v_2 v_3 W_1 W_2 W_3 + v_1 g_N^2 (\tau_{go} + \tau_{gg} + \tau_{gh} + \tau_{gf}) W_1^{n_1 + \frac{1}{2}} \frac{4\eta^2}{\theta_4^2} \\ & + v_3 h_N^2 (\tau_{ho} + \tau_{hg} + \tau_{hh} + \tau_{hf}) W_3^{n_3 + \frac{1}{2}} \frac{4\eta^2}{\theta_4^2} + v_2 f_N^2 (\tau_{fo} + \tau_{fg} + \tau_{fh} + \tau_{ff}) W_2^{n_2 + \frac{1}{2}} \frac{4\eta^2}{\theta_4^2} \} . \quad (6.5) \end{aligned}$$

The $4\eta^2/\theta_4^2$ factors in (6.5) originate from four of the internal bosonic coordinates, actually twisted in three different tori, that we do not distinguish for brevity while, for instance, the superscript in $W_1^{n_1 + 1/2}$ indicates that the corresponding winding sum is shifted in some direction by 1/2 unit. In the same notation, the direct-channel annulus amplitude then reads (in our convention, in A and M the momentum sum P^e , for instance, actually describes all integer levels)

$$\begin{aligned} A = \frac{1}{8} \{ & I_N^2 (\tau_{oo} + \tau_{og} + \tau_{oh} + \tau_{of}) P_1^e P_2^e P_3^e + g_N^2 (\tau_{oo} + \tau_{og} - \tau_{oh} - \tau_{of}) (-1)^{m_1} P_1^e \frac{4\eta^2}{\theta_2^2} \\ & + h_N^2 (\tau_{oo} - \tau_{og} + \tau_{oh} - \tau_{of}) (-1)^{m_3} P_3^e \frac{4\eta^2}{\theta_2^2} + f_N^2 (\tau_{oo} - \tau_{og} - \tau_{oh} + \tau_{of}) (-1)^{m_2} P_2^e \frac{4\eta^2}{\theta_2^2} \} . \quad (6.6) \end{aligned}$$

As in the previous cases, the transverse-channel Möbius amplitude is determined by the characters common to \tilde{K} and \tilde{A} . Thus, only the I_N charge contributes to \tilde{M} , while

the correct particle interpretation in the direct channel fixes some sign freedom. The final expression is

$$\begin{aligned} \tilde{M} = & -\frac{1}{4}I_N\{(\hat{\tau}_{oo} + \hat{\tau}_{og} + \hat{\tau}_{oh} + \hat{\tau}_{of})v_1v_2v_3W_1^eW_2^eW_3^e - v_1(\hat{\tau}_{oo} + \hat{\tau}_{og} - \hat{\tau}_{oh} - \hat{\tau}_{of})W_1^o\frac{4\hat{\eta}^2}{\hat{\theta}_2^2} \\ & - v_3(\hat{\tau}_{oo} - \hat{\tau}_{og} + \hat{\tau}_{oh} - \hat{\tau}_{of})W_3^o\frac{4\hat{\eta}^2}{\hat{\theta}_2^2} - v_2(\hat{\tau}_{oo} - \hat{\tau}_{og} - \hat{\tau}_{oh} + \hat{\tau}_{of})W_2^o\frac{4\hat{\eta}^2}{\hat{\theta}_2^2}\} , \end{aligned} \quad (6.7)$$

and the P-matrix in ref. [5] determines the direct channel amplitude

$$\begin{aligned} M = & -\frac{1}{8}I_N\{(\hat{\tau}_{oo} + \hat{\tau}_{og} + \hat{\tau}_{oh} + \hat{\tau}_{of})P_1^eP_2^eP_3^e + (\hat{\tau}_{oo} + \hat{\tau}_{og} - \hat{\tau}_{oh} - \hat{\tau}_{of})(-1)^{m_1}P_1^e\frac{4\hat{\eta}^2}{\hat{\theta}_2^2} \\ & + (\hat{\tau}_{oo} - \hat{\tau}_{og} + \hat{\tau}_{oh} - \hat{\tau}_{of})(-1)^{m_3}P_3^e\frac{4\hat{\eta}^2}{\hat{\theta}_2^2} + (\hat{\tau}_{oo} - \hat{\tau}_{og} - \hat{\tau}_{oh} + \hat{\tau}_{of})(-1)^{m_2}P_2^e\frac{4\hat{\eta}^2}{\hat{\theta}_2^2}\} . \end{aligned} \quad (6.8)$$

The proper Chan-Paton charge parametrization is in this case

$$\begin{aligned} I_N = n_o + n_g + n_h + n_f \quad , \quad g_N = n_o + n_g - n_h - n_f , \\ h_N = n_o - n_g + n_h - n_f \quad , \quad f_N = n_o - n_g - n_h + n_f , \end{aligned} \quad (6.9)$$

while the tadpole conditions give $I_N = 32$, so that the resulting D9 gauge group is $SO(n_o) \times SO(n_g) \times SO(n_h) \times SO(n_f)$. The massless spectrum is $N = 1$ supersymmetric, and contains, in addition to the vector multiplets, chiral multiplets in the representations $(\mathbf{n}_o, \mathbf{n}_g, \mathbf{1}, \mathbf{1})$, $(\mathbf{1}, \mathbf{1}, \mathbf{n}_h, \mathbf{n}_f)$, $(\mathbf{n}_o, \mathbf{1}, \mathbf{n}_h, \mathbf{1})$, $(\mathbf{1}, \mathbf{n}_g, \mathbf{1}, \mathbf{n}_f)$, $(\mathbf{n}_o, \mathbf{1}, \mathbf{1}, \mathbf{n}_f)$ and $(\mathbf{1}, \mathbf{n}_g, \mathbf{n}_h, \mathbf{1})$. This model can be seen as a discrete deformation of the $N = 4$ supersymmetric type I model with a Wilson line that breaks the gauge group $SO(32)$ to $SO(n_o) \times SO(n_g) \times SO(n_h) \times SO(n_f)$. In the particular case $n_g = n_h = n_f = 0$ one recovers again $SO(32)$, as in the $N = 4 \rightarrow N = 2$ example of Section 4. This model is also dual to a heterotic vacuum with $N = 4$ supersymmetry spontaneously broken to $N = 1$ by compactification [11], as can be seen by arguments similar to those in [1].

7. Conclusions

In this paper we have constructed different models with partial and total supersymmetry breaking in 6d, 5d and 4d type I strings, starting from type IIB models and deriving by standard methods [3, 4, 5, 17] the corresponding open descendants. In the partial breaking case, the resulting models provide new $N = 2$ and $N = 1$ supersymmetric type I compactifications, that may also be related by duality arguments to other types of vacua. In particular, the M-theory breaking model discussed in Section 5 has a natural interpretation in terms of the eleventh dimension of M-theory [15, 16], while the Scherk-Schwarz breaking models discussed in Sections 4 and 6 are easily interpreted as perturbative breakings in the dual heterotic language.

The models present some surprising features. Thus, for instance, in the type I' context the model of Section 5 contains D8 and D4 branes, both having as origin in the M-theory picture the Horava-Witten 10d boundaries. As another example, in the $N = 4 \rightarrow N = 1$ model of Section 6, supersymmetry breaking removes from the Klein bottle the tadpole contributions that in the supersymmetric $Z_2 \times Z_2$ type I model are related to D5 branes, and the resulting spectrum has thus no D5 branes. More general models contain richer patterns of D5-brane configurations. It would be interesting to construct the corresponding effective field theories and to compare them with the existing field theory models of partial supersymmetry breaking [22].

We have provided additional evidence that the general rule in these constructions is that branes with world-volume parallel to the breaking coordinate feel supersymmetry breaking at tree-level, while branes orthogonal to this coordinate have a massless spectrum that at tree-level is still supersymmetric. In these sectors, the breaking propagates through radiative corrections and the resulting picture has some potential applications that are interesting for phenomenology, for instance to models with a low string scale, that have received some attention lately [23], or to models with the “world as a brane” picture [24]. It

would be interesting to study more general 4d examples with chirality [25, 18] and $N = 1$ supersymmetry spontaneously broken by Scherk-Schwarz compactifications based on R-symmetries and to investigate whether the chirality flip observed in the temperature-like $(-1)^F$ breaking applies to these cases as well. Finally, a detailed evaluation of the radiative corrections to some relevant quantities in models with massless sectors where supersymmetry is unbroken at tree level would provide further insight on the less conventional M-theory breaking mechanism.

Acknowledgements G.D., E.D. and A.S. are grateful to the Centre de Physique Théorique of the Ecole Polytechnique for the kind hospitality during the course of this work. I.A. would like to acknowledge interesting discussions with C. Angelantonj, K. Benakli and K. Foerger. Finally, G.D. would like to thank the Ecole Polytechnique for a CIES Fellowship.

References

- [1] I. Antoniadis, E. Dudas and A. Sagnotti, *Nucl. Phys.* **B544** (1999) 469, hep-th/9807011.
- [2] C. Bachas, hep-th/9503030;
J. Blum and K.R. Dienes, *Phys. Lett.* **B414** (1997) 260, *Nucl. Phys.* **B516** (1998) 83.
- [3] A. Sagnotti, in: Cargese '87, Non-Perturbative Quantum Field Theory, eds. G. Mack et al. (Pergamon Press, Oxford, 1988) p. 521.
- [4] G. Pradisi and A. Sagnotti, *Phys. Lett.* **B216** (1989) 59.
- [5] M. Bianchi and A. Sagnotti, *Phys. Lett.* **B247** (1990) 517, *Nucl. Phys.* **B361** (1991) 519.
- [6] P. Horava, *Nucl. Phys.* **B327** (1989) 461.

- [7] J. Scherk and J.H. Schwarz, *Nucl. Phys.* **B153** (1979) 61, *Phys. Lett.* **B82** (1979) 60;
P. Fayet, *Phys. Lett.* **B159** (1985) 121, *Nucl. Phys.* **B263** (1986) 649.
- [8] R. Rohm, *Nucl. Phys.* **B237** (1984) 553.
- [9] C. Kounnas and M. Porrati, *Nucl. Phys.* **B310** (1988) 355;
S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner, *Nucl. Phys.* **B318** (1989) 75;
C. Kounnas and B. Rostand, *Nucl. Phys.* **B341** (1990) 641;
I. Antoniadis and C. Kounnas, *Phys. Lett.* **B261** (1991) 369.
- [10] I. Antoniadis, *Phys. Lett.* **B246** (1990) 377;
Proc. PASCOS-91 Symposium, Boston (World Scientific, Singapore, 1991), p. 718;
K. Benakli, *Phys. Lett.* **B386** (1996) 106.
- [11] E. Kiritsis and C. Kounnas, *Nucl. Phys.* **B503** (1997) 117.
- [12] C. Vafa and E. Witten, *Nucl. Phys. Proc. Suppl.* **46** (1996) 225.
- [13] J. Polchinski and E. Witten, *Nucl. Phys.* **B460** (1996) 525.
- [14] P. Horava and E. Witten, *Nucl. Phys.* **B460** (1996) 506, **B475** (1996) 94.
- [15] I. Antoniadis and M. Quirós, *Phys. Lett.* **B392** (1997) 61,
Nucl. Phys. **B505** (1997) 109, *Phys. Lett.* **B416** (1998) 327.
- [16] E. Dudas and C. Grojean, *Nucl. Phys.* **B507** (1997) 553;
E. Dudas, *Phys. Lett.* **B416** (1998) 309.
- [17] E. Gimon and J. Polchinski, hep-th/9601038.
- [18] G. Aldazabal, A. Font, L.E. Ibáñez and G. Violero, hep-th/9804026;
Z. Kakushadze, G. Shiu and S.-H.H. Tye, hep-th/9804092.

- [19] M. Bianchi, G. Pradisi and A. Sagnotti, *Nucl. Phys.* **B376** (1992) 365;
M. Bianchi, *Nucl. Phys.* **B528** (1998) 73;
E. Witten, *J. High Energy Phys.* **02** (1998) 006;
Z. Kakushadze, G. Shiu and S.-H.H. Tye, *Phys. Rev.* **D58** (1998) 086001;
M. Bianchi, E. Gava, F. Morales and K.S. Narain, hep-th/9811013.
- [20] G. Pradisi, A. Sagnotti and Ya.S. Stanev, *Phys. Lett.* **B354** (1995) 279,
B356 (1995) 230, **B381** (1996) 97.
- [21] M. Bianchi, Ph.D. Thesis, preprint ROM2F-92/13.
- [22] S. Ferrara, L. Girardello and M. Porrati, *Phys. Lett.* **B366** (1996) 155;
I. Antoniadis, H. Partouche and T.R. Taylor, *Phys. Lett.* **B372** (1996) 83;
J. Bagger and A. Galperin, *Phys. Lett.* **B412** (1997) 296, *Phys. Rev.* **D55** (1997) 1091.
- [23] J.D. Lykken, *Phys. Rev.* **D54** (1996) 3693;
N. Arkani-Hamed, S. Dimopoulos and G. Dvali, hep-ph/9803315, hep-ph/9807344;
K.R. Dienes, E. Dudas and T. Gherghetta, hep-ph/9803466, hep-ph/9806292;
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, hep-ph/9804398;
G. Shiu and S.-H.H. Tye, hep-th/9805157; R. Sundrum, hep-ph/9805471;
C. Bachas, hep-ph/9807415;
Z. Kakushadze and S.-H.H. Tye, hep-th/9809147; K. Benakli, hep-ph/9809582;
I. Antoniadis, S. Dimopoulos, A. Pomarol and M. Quiros, hep-ph/9810410;
C.P. Burgess, L.E. Ibáñez and F. Quevedo, hep-ph/9810535.
- [24] A. Lukas, B.A. Ovrut, K.S. Stelle and D. Waldram, hep-th/9803235;
J. Lykken, E. Poppitz and S. Trivedi, hep-th/9806080;
L. Randall and R. Sundrum, hep-th/9810155.
- [25] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Ya. S. Stanev,
Phys. Lett. **B385** (1996) 96;

- Z. Kakushadze, *Nucl. Phys.* **B512** (1998) 221;
Z. Kakushadze and G. Shiu, *Phys. Rev.* **D56** (1997) 3686,
Nucl. Phys. **B520** (1998) 75;
G. Zwart, hep-th/9708040;
C. Angelantonj, hep-th/9810214.